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Canonical Gauge Coupling Unification in the Standard Model with High-Scale Supersymmetry Breaking

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Abstract

Inspired by the string landscape and the unified gauge coupling relation in the F-theory Grand Unified Theories (GUTs) and GUTs with suitable high-dimensional operators, we study the canonical gauge coupling unification and Higgs boson mass in the Standard Model (SM) with high-scale supersymmetry breaking. In the SM with GUT-scale supersymmetry breaking, we achieve the gauge coupling unification at about 5.3×10^{13} GeV, and the Higgs boson mass is predicted to range from 130 GeV to 147 GeV. In the SM with supersymmetry breaking scale from 10^4 GeV to 5.3×10^{13} GeV, gauge coupling unification can always be realized and the corresponding GUT scale M_U is from 10^{16} GeV to 5.3×10^{13} GeV, respectively. Also, we obtain the Higgs boson mass from 114.4 GeV to 147 GeV. Moreover, the discrepancies among the SM gauge couplings at the GUT scale are less than about 4-6%. Furthermore, we present the $SU(5)$ and $SO(10)$ models from the F-theory model building and orbifold constructions, and show that we do not have the dimension-five and dimension-six proton decay problems even if $M_U \leq 5 \times 10^{15}$ GeV.

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I. INTRODUCTION

It is well-known that there might exist an enormous "landscape" for long-lived metastable string/M theory vacua where the moduli can be stabilized and supersymmetry may be broken in the string models with flux compactifications [1]. Applying the "weak anthropic principle" [2], the string landscape proposal might provide the first concrete solution to the cosmological constant problem, and it may address the gauge hierarchy problem in the Standard Model (SM). Notably, the supersymmetry breaking scale can be high if there exist many supersymmetry breaking parameters or many hidden sectors [3, 4]. Although there is no definite conclusion whether the string landscape predicts high-scale or TeV-scale supersymmetry breaking [3], it is interesting to study the models with high-scale supersymmetry breaking due to the turn on of the Large Hadron Collider (LHC) [4–8].

Assuming that supersymmetry is indeed broken at a high scale, we can classify the supersymmetry breaking scale as follows [5]: (1) the string scale or grand unification scale; (2) an intermediate scale; and (3) the TeV scale. We do not consider the TeV-scale supersymmetry here since it has been studied extensively during the last thirty years. However, we would like to emphasize that for high-scale supersymmetry breaking, most of the problems associated with some low energy supersymmetric models, for example, excessive flavor and CP violations, dimension-five fast proton decay and the stringent constraints on the lightest CP-even neutral Higgs boson mass, may be solved automatically.

If supersymmetry is broken at the high scale, the minimal model at the low energy is the Standard model. The SM explains existing experimental data very well, including electroweak precision tests. Moreover, we can easily incorporate aspects of physics beyond the SM through small variations, for example, dark matter, dark energy, atmospheric and solar neutrino oscillations, baryon asymmetry, and inflation [9]. Also, the SM fermion masses and mixings can be explained via the Froggatt-Nielsen mechanism [10]. However, there are still some limitations of the SM, for example, the lack of explanation of gauge coupling unification and charge quantization [6, 7].

Charge quantization can easily be realized by embedding the SM into the Grand Unified Theories (GUTs). Anticipating that the Higgs particle might be the only new physics observed at the LHC, thus confirming the SM as the low energy effective theory, we should reconsider gauge coupling unification in the SM. Previously, the generic gauge coupling

unification can be defined by

$$k_Y g_Y^2 = g_2^2 = g_3^2 ~, \tag{1}$$

where k_Y is the normalization constant for the $U(1)_Y$ hypercharge interaction, and g_Y, g_2, and g_3 are the gauge couplings for the $U(1)_Y$, $SU(2)_L$, and $SU(3)_C$ gauge groups, respectively. However, it is well-known that gauge coupling unification cannot be achieved in the SM with canonical $U(1)_Y$ normalization, *i.e.*, the Georgi-Glashow $SU(5)$ normalization with $k_Y = 5/3$ [11]. Interestingly, it was shown that gauge coupling unification can be realized in the non-canonical $U(1)_Y$ normalization with $k_Y = 4/3$ [6, 7]. The orbifold GUTs with such $U(1)_Y$ normalization have been constructed as well. The key question remains: can we realize the gauge coupling unification in the SM with canonical $U(1)_Y$ normalization?

During the last a few years, GUTs have been constructed locally in the F-theory model building [12–21]. A brand new feature is that the $SU(5)$ gauge symmetry can be broken down to the SM gauge symmetry by turning on $U(1)_Y$ flux [14, 15, 21], and the $SO(10)$ gauge symmetry can be broken down to the $SU(5) \times U(1)_X$ and $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ gauge symmetries by turning on the $U(1)_X$ and $U(1)_{B-L}$ fluxes, respectively [14, 15, 17, 18, 20, 21]. It has been shown that the gauge kinetic functions receive the corrections from $U(1)$ fluxes [16, 19–21]. In particular, in the $SU(5)$ models with $U(1)_Y$ flux [16, 19] and in the $SO(10)$ models with $U(1)_{B-L}$ flux [21], the SM gauge couplings at the GUT scale satisfy the following condition

$$\frac{1}{\alpha_2} - \frac{1}{\alpha_3} ~=~ \frac{5}{3}\left(\frac{1}{\alpha_1} - \frac{1}{\alpha_3}\right) ~, \tag{2}$$

where $\alpha_1 = 5\alpha_Y/3$, $\alpha_Y = g_Y^2/4\pi$, and $\alpha_j = g_j^2/4\pi$ for $j = 2,~3$. In other words, the gauge coupling unification scale M_U is defined by Eq. (2). Especially, we have canonical $U(1)_Y$ normalization here. Moreover, the above gauge coupling relation at the GUT scale can be realized in the four-dimensional GUTs with suitable high-dimensional operators [22–25] and in the orbifold GUTs [26–32] with similar high-dimensional operators on the 3-branes at the fixed points where the complete GUT gauge symmetries are preserved. We emphasize that the above gauge coupling relation at the GUT scale was first given in Ref. [24].

In this paper, considering high-scale supersymmetry breaking inspired by the string landscape, we shall study the gauge coupling unification in the SM where the GUT-scale gauge coupling relation is given by Eq. (2). In the SM with GUT-scale supersymmetry breaking,

the SM gauge couplings are unified at about 5.3×10^{13} GeV. In the SM with supersymmetry breaking scale from 10^4 GeV to 5.3×10^{13} GeV, gauge coupling unification can always be realized, and we obtain the corresponding GUT scale M_U from 10^{16} GeV to 5.3×10^{13} GeV, respectively. Also, the discrepancies among the SM gauge couplings at the GUT scale are less than about 4-6%. Moreover, we calculate the SM Higgs boson mass. In the SM with GUT-scale supersymmetry breaking, the Higgs boson mass is predicted to range from 130 GeV to 147 GeV. And in the SM with supersymmetry breaking scale from 10^4 GeV to 5.3×10^{13} GeV, we obtain the Higgs boson mass from 114.4 GeV to 147 GeV where the low bound on the SM Higgs boson mass from the LEP experiment [33] has been included. Furthermore, we present the $SU(5)$ and $SO(10)$ models from the F-theory model building and orbifold constructions, and show that there are no dimension-five and dimension-six proton decay problems even if $M_U \leq 5\times10^{15}$ GeV.

This paper is organized as follows. In Section II, we study the gauge coupling unification in the SM with high-scale supersymmetry breaking. In Section III, we consider the Higgs boson masses. We present the concrete $SU(5)$ and $SO(10)$ models without proton decay problems in Section IV. And our conclusion is given in Section V.

II. GAUGE COUPLING UNIFICATION

For simplicity, we consider the universal high-scale supersymmetry breaking. Above the universal supersymmetry breaking scale M_S, we consider the supersymmetric SM. Following the procedures in Ref. [7] where all the relevant renormalization group equations (RGEs) are given, we consider the two-loop RGE running for the SM gauge couplings, and one-loop RGE running for the SM fermion Yukawa couplings.

In numerical calculations, we choose the top quark pole mass $M_t = 173.1 \pm 1.3$ GeV [34], and the strong coupling constant $\alpha_3(M_Z) = 0.1184 \pm 0.0007$ [35], where M_Z is the Z boson mass. Also, the fine structure constant α_{EM}, weak mixing angle θ_W and Higgs vacuum expectation value (VEV) v at M_Z are taken as follows [35]

$$\alpha_{EM}^{-1}(M_Z) = 128.91\,, \quad \sin^2\theta_W(M_Z) = 0.23116\,, \quad v = 174.10\,\mathrm{GeV}\,. \tag{3}$$

First, we consider the GUT-scale universal supersymmetry breaking, *i.e.*, we only have the SM below the GUT scale. With the GUT-scale gauge coupling relation in Eq. (2),

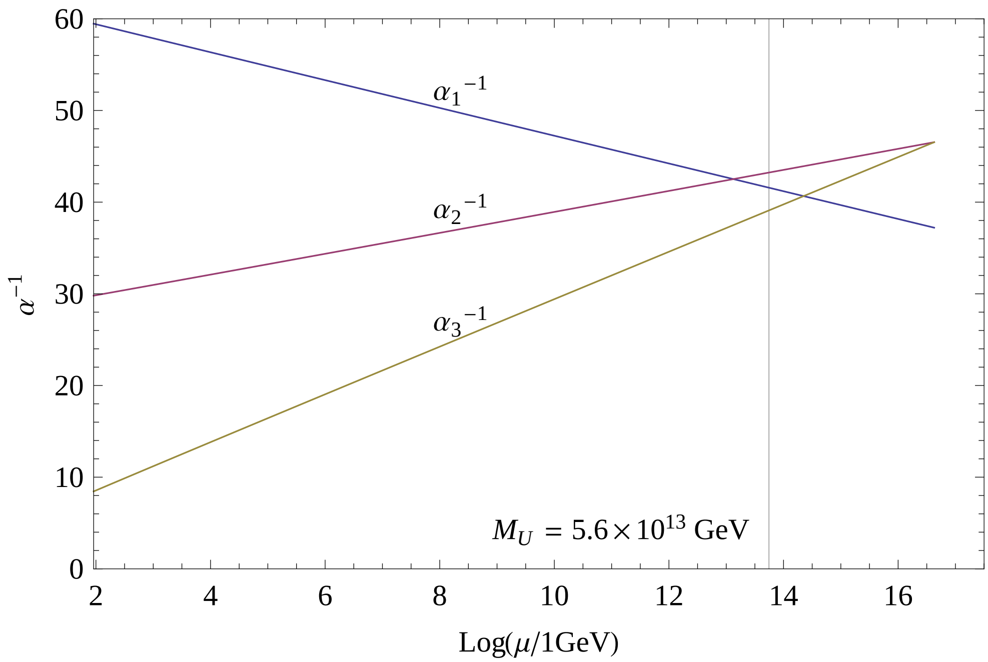

FIG. 1: Canonical gauge coupling unification in the SM where the gauge coupling unification scale M_U is defined by Eq. (2).

we present the gauge coupling unification in Fig. 1, and find that the unification scale is about 5.3×10^{13} GeV. Next, we consider the intermediate-scale universal supersymmetry breaking. Interestingly, gauge coupling unification can always be realized. In Fig. 2, we present the GUT scale for the universal supersymmetry breaking scale M_S from 10^4 GeV to 5.3×10^{13} GeV. The GUT scale decreases when the supersymmetry breaking scale increases. Moreover, the GUT scale varies from 10^{16} GeV to 5.3×10^{13} GeV for the supersymmetry breaking scale from 10^4 GeV to 5.3×10^{13} GeV, respectively. Moreover, the GUT scale is almost independent on the mixing parameter $\tan\beta$, which is defined in the first paragraph in the next Section.

To demonstrate that the deviations from the complete gauge coupling universality are still modest, we study the discrepancies among the SM gauge couplings at the GUT scale by defining two parameters δ_+ and δ_- at the GUT scale

$$\delta_+ = \frac{\alpha_2^{-1} - \alpha_1^{-1}}{\alpha_1^{-1}} ~, \qquad\qquad \delta_- = \frac{\alpha_3^{-1} - \alpha_1^{-1}}{\alpha_1^{-1}} ~. \tag{4}$$

In Fig. 3, we present δ_+ and δ_- for the supersymmetry breaking scale from 10^4 GeV to 5.3×10^{13} GeV. We find that δ_+ and $|\delta_-|$ increase when the supersymmetry breaking scale M_S increases. Also, δ_+ and $|\delta_-|$ are smaller than 4% and 6%, respectively. Similar to the

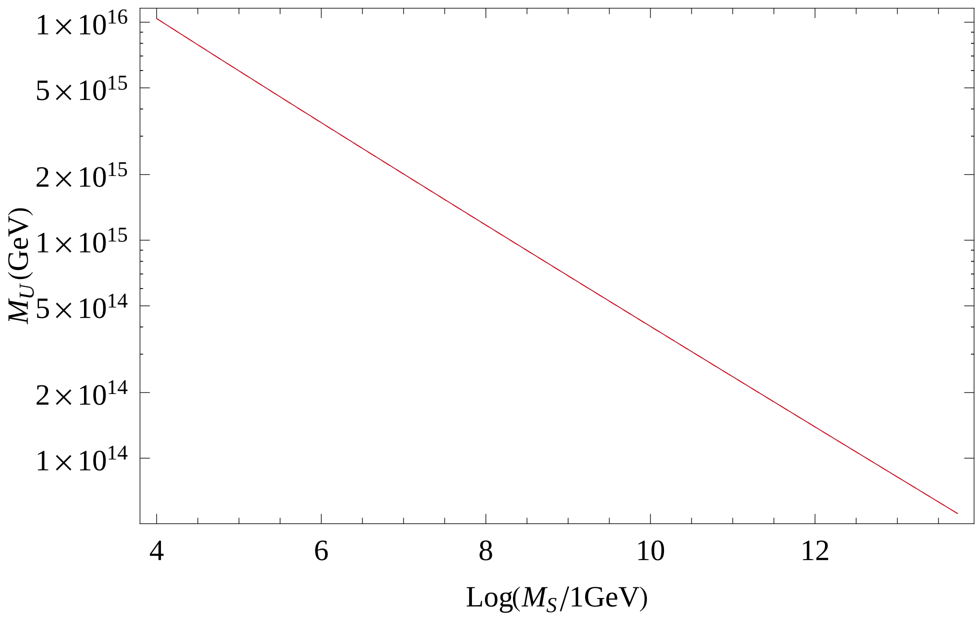

FIG. 2: The GUT scale M_U versus the universal supersymmetry breaking scale M_S. We consider $\tan\beta = 3$ (dotted line) and 35 (solid line), and $M_t = 171.8$ GeV, 173.1 GeV, 174.4 GeV. The results for different cases are roughly the same.

GUT scale, δ_+ and δ_- are almost independent on $\tan\beta$ as well. Thus, these discrepancies among the SM gauge couplings at the GUT scale are indeed small.

III. HIGGS BOSON MASS

If the Higgs particle is the only new physics discovered at the LHC and then the SM is confirmed as the low energy effective theory, the Higgs boson mass is one of the most important parameters. Above the supersymmetry breaking scale, we have supersymmetric SMs. There generically exists one pair of Higgs doublets H_u and H_d, which give masses to the up-type quarks and down-type quarks/charged leptons, respectively. Below the supersymmetry breaking scale, we only have the SM. Let us define the SM Higgs doublet H as $H \equiv -\cos\beta i\sigma_2 H_d^* + \sin\beta H_u$, where σ_2 is the second Pauli matrix and $\tan\beta$ is a mixing parameter [4–6]. For simplicity, we assume the gauginos, squarks, Higgsinos, and the other combination of the scalar Higgs doublets $\sin\beta i\sigma_2 H_d^* + \cos\beta H_u$ have the universal supersymmetry breaking soft mass M_S. We first assume that supersymmetry is broken at the GUT scale M_U, *i.e.*, $M_S \simeq M_U$. And then we assume that supersymmetry is broken at the

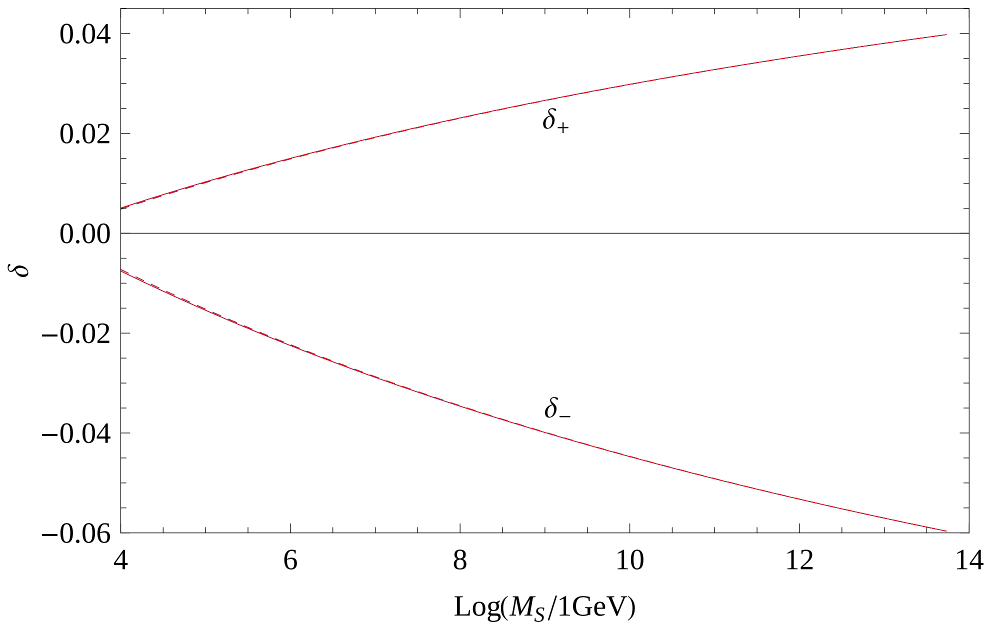

FIG. 3: δ_+ and δ_- versus the universal supersymmetry breaking scale M_S. We consider $\tan\beta =$ 3 (dotted line) and 35 (solid line), and $M_t = 171.8$ GeV, 173.1 GeV, and 174.4 GeV. The results for different cases are roughly the same.

intermediate scale, *i.e.*, below the GUT scale but higher than the electroweak scale, such as between 10^4 GeV and M_U.

We consider the supersymmetry breaking scale M_S from 10^4 GeV to the SM unification scale 5.3×10^{13} GeV. At the supersymmetry breaking scale, we can calculate the Higgs boson quartic coupling λ [4–6]

$$\lambda(M_S) = \frac{g_1^2(M_S) + k_Y g_2^2(M_S)}{4k_Y} \cos^2 2\beta, \tag{5}$$

where $k_Y = 5/3$, and then evolve it down to the Higgs boson mass scale. The one-loop RGE for the quartic coupling is given in Ref. [7] as well. To predict the SM Higgs boson mass, we consider the two-loop RGE running for the SM gauge couplings, and one-loop RGE running for the SM fermion Yukawa couplings and Higgs quartic coupling. Using the one-loop effective Higgs potential with top quark radiative corrections, we calculate the Higgs boson mass by minimizing the effective potential

$$V_{eff} = m_h^2 H^\dagger H + \frac{\lambda}{2!}(H^\dagger H)^2 - \frac{3}{16\pi^2} h_t^4 (H^\dagger H)^2 \left[\log \frac{h_t^2(H^\dagger H)}{Q^2} - \frac{3}{2}\right], \tag{6}$$

where m_h^2 is the squared Higgs boson mass, h_t is the top quark Yukawa coupling from $m_t = h_t v$, and the scale Q is chosen to be at the Higgs boson mass. For the $\overline{MS}$ top quark

mass m_t, we use the two-loop corrected value, which is related to the top quark pole mass M_t by [36]

$$M_t = m_t(m_t)\left\{1 + \frac{4\alpha_3(m_t)}{3\pi} + \left[13.4434 - 1.0414\sum_{k=1}^{5}(1 - \frac{4}{3}\frac{m_k}{m_t})\right]\left[\frac{\alpha_3(m_t)}{\pi}\right]^2\right\} , \quad (7)$$

where m_k denotes the other quark mass. Also, the two-loop RGE running for α_3 has been used.

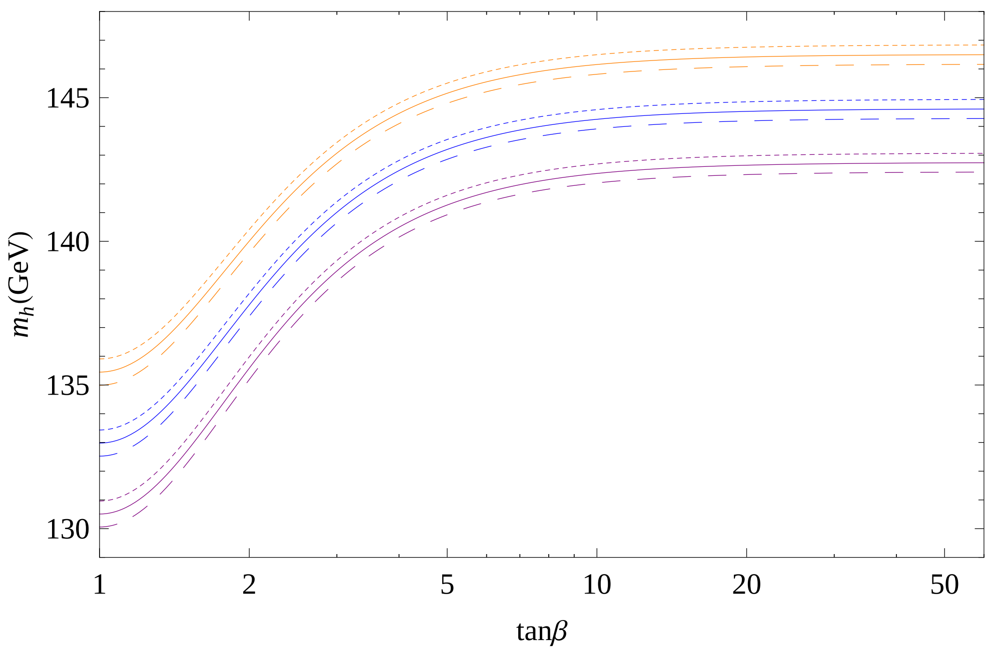

FIG. 4: The predicted Higgs boson mass versus $\tan\beta$ in the SM with GUT scale supersymmetry breaking. The top (orange) three curves are for $M_t + \delta M_t$, the bottom (purple) $M_t - \delta M_t$, and the middle (blue) M_t. The dotted curves are for $\alpha_3 - \delta\alpha_3$, the dash ones for $\alpha_3 + \delta\alpha_3$, and the solid ones for α_3. Here, we choose $M_t = 173.1$ GeV and $\delta M_t = 1.3$ GeV.

For the SM with GUT-scale supersymmetry breaking, the predicted Higgs boson mass is shown as a function of $\tan\beta$ for different M_t and α_3 in Fig. 4. When we increase top quark mass or decrease strong coupling, the predicted Higgs boson mass will increase. If we vary M_t and α_s within their 1σ range, and $\tan\beta$ from 1 to 60, the predicted Higgs boson mass will range from 130 GeV to 147 GeV. Moreover, focussing on the high-scale supersymmetry breaking around 10^{14} GeV, Hall and Nomura made a very fine prediction for the Higgs boson mass from 128 GeV to 141 GeV [8]. Thus, our predicted Higgs boson masses are a

little bit larger than their results. Concretely speaking, the discrepancy between our low bound and their low bound is about 1.5% while the discrepancy between our upper bound and their upper bound is about 4%. Because the inputs for the top quark mass are the same, it seems to us that these discrepancies may be due to the following three reasons: (1) Our supersymmetry breaking scale is 5.3×10^{13} GeV while their supersymmetry breaking scale is 4×10^{14} GeV, thus, the boundary conditions are different. (2) For the SM fermion Yukawa couplings and Higgs quartic coupling, we consider the one-loop RGE running while they considered the two-loop RGE running. (3) We consider $\tan\beta$ from 1 to 60 while they considered $\tan\beta$ from 1 to 10. Although each of these effects is small, we may understand the discrepancies by summing up all these effects.

In Fig. 5, we present the Higgs boson mass for the intermediate-scale supersymmetry breaking. Generically, the predicted Higgs boson mass will increase when supersymmetry breaking scale increases. For supersymmetry breaking scale M_S varying from 10^4 GeV to 5.3×10^{13} GeV, and $\tan\beta$ between 3 and 35, M_t within its 1σ range, the predicted Higgs boson mass will range from 114.4 GeV to 146 GeV, where the low bound on the SM Higgs boson mass from the LEP experiment [33] has been included. If we also vary α_3 within its 1σ range, the predicted Higgs boson mass will range from 114.4 GeV to 147 GeV.

IV. F-THEORY GUTS AND ORBIFOLD GUTS

Because the GUT scale in our models can be as small as 5.3×10^{13} GeV, we might have dimension-five and dimension-six proton decay problems. In this paper, we shall consider the $SU(5)$ and $SO(10)$ models from the local F-theory constructions and the orbifold constructions, where these proton decay problems can be solved. In particular, the GUT-scale gauge coupling relation given by Eq. (2) can be realized.

Let us explain our convention. In the supersymmetric SMs, we denote the left-handed quark doublets, right-handed up-type quarks, right-handed down-type quarks, left-handed lepton doublets, right-handed neutrinos, and right-handed charged leptons as Q_i, U_i^c, D_i^c, L_i, N_i^c, and E_i^c, respectively. In the $SU(5)$ models, the SM fermions form $\mathbf{10}_i = (Q_i,\ U_i^c,\ E_i^c)$ and $\overline{\mathbf{5}}_i = (D_i^c,\ L_i)$ and $\mathbf{1}_i = N_i^c$ representations. The Higgs fields form $\mathbf{5}_H = (T_u,\ H_u)$ and $\overline{\mathbf{5}}_{\overline{H}} = (T_d,\ H_d)$ representations, where T_u and T_d are the colored Higgs fields. In the $SO(10)$ models, one family of the SM fermions form a spinor $\mathbf{16}_i$ representation, and all the Higgs

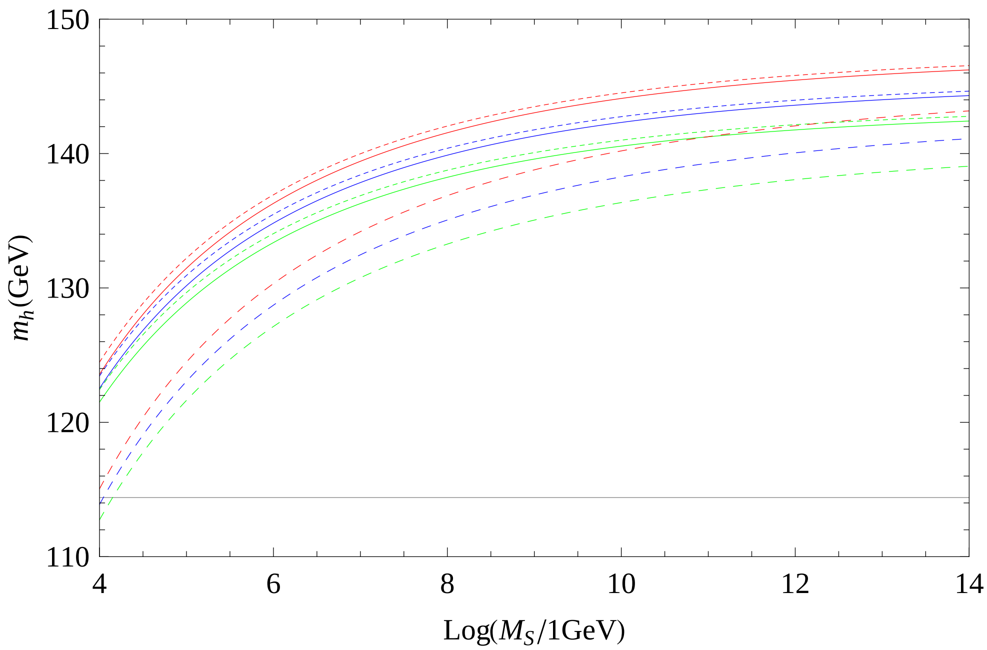

FIG. 5: The predicted Higgs boson mass versus M_S in the SM with high-scale supersymmetry breaking. The top (red) two curves are for $M_t + \delta M_t$, the bottom (green) $M_t - \delta M_t$, and the middle (blue) M_t. The dash curves are for $\tan\beta = 3$, the solid ones for $\tan\beta = 10$, and the dotted ones for $\tan\beta = 35$. The horizontal line is the LEP low bound 114.4 GeV.

fields form a $\mathbf{10}_H = (T_u,\ H_u,\ T_d,\ H_d)$ representation.

First, we briefly review the proton decay. The dimension-five proton decays arise from the color-Higgsino exchanges. In $SU(5)$ and $SO(10)$ models, we have the following superpotential in terms of the SM fermions

$$W \ = \ y_u^{ij}(Q_iQ_j + 2U_i^cE_j^c)T_u + y_{de}^{ij}(2Q_iL_j + 2U_i^cD_j^c)T_d + M_T T_u T_d \ , \tag{8}$$

where y_u^{ij} are the Yukawa couplings for the up-type quarks, and y_{de}^{ij} are the Yukawa couplings for the down-type quarks and charged leptons. In $SO(10)$ models, we shall have $y_u^{ij} = y_{de}^{ij}$ as well. The dimension-five proton decay operators are obtained after we integrate out the heavy colored Higgs fields T_u and T_d. The corresponding proton partial lifetime from dimension-five proton decay is proportional to $M_T^2 M_S^2$, and we require $M_T M_S \ \geq \ 10^{20}$ GeV^2 from the current experimental bounds [37, 38].

The dimension-six proton decay operators are obtained after we integrate out the heavy gauge boson fields. In $SU(5)$ models, we have two kinds of operators $\mathbf{10}^*{}_i\mathbf{10}_i\mathbf{10}^*{}_j\mathbf{10}_j$ and $\mathbf{10}^*{}_i\mathbf{10}_i\mathbf{5}^*{}_j\mathbf{5}_j$. In the flipped $SU(5)\times U(1)_X$ models, we also have two kinds of operators

$(\mathbf{10},\mathbf{1})^*{}_i(\mathbf{10},\mathbf{1})_i(\mathbf{10},\mathbf{1})^*{}_j(\mathbf{10},\mathbf{1})_j$ and $(\mathbf{10},\mathbf{1})^*{}_i(\mathbf{10},\mathbf{1})_i(\overline{\mathbf{5}},-\mathbf{3})^*{}_j(\overline{\mathbf{5}},-\mathbf{3})_j$. In $SO(10)$ models, we only have one kind of operators $\mathbf{16}^*{}_i\mathbf{16}_i\mathbf{16}^*{}_j\mathbf{16}_j$. In terms of the SM fields, we obtain the possible dimension-six operators which contribute to the proton decay [39]

$$O_I = \frac{g_U^2}{2M_{(X,Y)}^2} \overline{U_i^c} \gamma^\mu Q_i \overline{E_j^c} \gamma_\mu Q_j , \tag{9}$$

$$O_{II} = \frac{g_U^2}{2M_{(X,Y)}^2} \overline{U_i^c} \gamma^\mu Q_i \overline{D_j^c} \gamma_\mu L_j , \tag{10}$$

$$O_{III} = \frac{g_U^2}{2M_{(X',Y')}^2} \overline{D_i^c} \gamma^\mu Q_i \overline{U_j^c} \gamma_\mu L_j , \tag{11}$$

$$O_{IV} = \frac{g_U^2}{2M_{(X',Y')}^2} \overline{D_i^C} \gamma^\mu Q_i \overline{N_j^c}_L \gamma_\mu Q_j , \tag{12}$$

where g_U is the unified gauge coupling at the GUT scale, and $M_{(X,Y)}$ and $M_{(X',Y')}$ are the masses of the superheavy gauge bosons in the $SU(5)$ models and flipped $SU(5) \times U(1)_X$ models, respectively. In the $SU(5)$ models, we obtain the effective operators O_I and O_{II} respectively in Eqs. (9) and (10) after the superheavy gauge fields $(X, Y) = (\mathbf{3}, \mathbf{2}, \mathbf{5/6})$ are integrated out. In the flipped $SU(5) \times U(1)_X$ models, we obtain the effective operators O_{III} and O_{IV} respectively in Eqs. (11) and (12) after the superheavy gauge fields $(X', Y') = (\mathbf{3}, \mathbf{2}, -\mathbf{1/6})$ are integrated out. Because both the $SU(5)$ models and the flipped $SU(5) \times U(1)_X$ models can be embedded into the $SO(10)$ models, we have all these superheavy gauge fields as well as all the above dimension-six proton decay operators. Note that the dimension-six proton decays have not been observed from the experiments, we obtain that the GUT scale is higher than about 5×10^{15} GeV. Because the GUT scale in our models can be as small as 5.3×10^{13} GeV, we require that the (X, Y) gauge bosons in the $SU(5)$ models and the (X, Y) and (X', Y') gauge bosons in the $SO(10)$ models do not generate the above dimension-six proton decay operators. Therefore, we need to forbid at least some of the couplings between the superheavy gauge fields and the SM fermions in the model building.

Second, let us consider the F-theory GUTs which do not have proton decay problem. In the F-theory $SU(5)$ model proposed in Ref. [21], the Higgs fields $\mathbf{5}_H = (T_u,\ H_u)$ and $\overline{\mathbf{5}}_{\overline{H}} = (T_d,\ H_d)$ are on the different Higgs curves, and T_u and T_d do not have zero modes by choosing proper $U(1)$ fluxes. And then the KK modes of T_u and T_d do not form vector-like particles, *i.e.*, the third term in Eq. (8) does not exist. The mass terms between the KK modes of T_u and T_d arise from the usual μ term. So the proton partial lifetime via the dimension-five proton decay is proportional to $M_{T_u}^2 M_{T_d}^2 M_S^2/\mu^2$. In generic GUTs with high-

scale supersymmetry breaking, we have $M_S \simeq \mu$, and $M_{T_u} \sim M_{T_d} \sim M_U$. Thus, the proton partial lifetime via the dimension-five proton decay is proportional to $M_U^2 \geq 10^{26}$ GeV^2, which is much larger than 10^{20} GeV^2. And then we do not have the dimension-five proton decay problem. Moreover, the SM quarks Q_i and U_i^c are on different matter curves. And then the X and Y gauge bosons can not couple to both Q_i and U_i^c. Therefore, we do not have the dimension-six proton decay problem via superheavy gauge boson exchanges.

In the Type I and Type II F-theory $SO(10)$ models proposed in Ref. [21] where the $SO(10)$ gauge symmetry is broken down to the $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ gauge symmetry, the SM fermions Q_i, E_i^c, and N_i^c are on one matter curve, while U_i^c, D_i^c, and L_i are on the other matter curve. On the Higgs $\mathbf{10}_H = (T_u,\ H_u,\ T_d,\ H_d)$ curve, T_u and T_d do not have zero modes by choosing proper $U(1)$ fluxes, and the KK modes of T_u and T_d do not form vector-like particles. Thus, similar to the discussions in the above F-theory $SU(5)$ models, we do not have the dimension-five proton decay problem. Moreover, the SM quarks Q_i and U_i^c/D_i^c are on different matter curves. So the X and Y gauge bosons can not couple to both Q_i and U_i^c, and the X' and Y' gauge bosons can not couple to both Q_i and D_i^c, Therefore, we do not have the dimension-six proton decay problem via superheavy gauge boson exchanges.

Third, we consider the five-dimensional orbifold $SU(5)$ and $SO(10)$ models on $S^1/(Z_2 \times Z_2')$ where the proton decay problems can be solved as well [26–32]. We assume that the fifth dimension is a circle S^1 with coordinate y and radius R. The orbifold $S^1/(Z_2 \times Z_2')$ is obtained by the circle S^1 moduloing the following equivalent classes

$$P: \quad y \sim -y ~, \qquad\qquad P': \quad y' \sim -y' ~, \tag{13}$$

where $y' = y + \pi R/2$. There are two inequivalent 3-branes located at the fixed points $y = 0$ and $y = \pi R/2$, which are denoted by O_B and O_B', respectively. In particular, the zero modes of the SM fermions in the bulk do not form the complete GUT representations due to the orbifold gauge symmetry breaking [32].

In the orbifold $SU(5)$ models (for a concrete example, see Ref. [28]), the $SU(5)$ gauge symmetry is broken down to the SM gauge symmetry via orbifold projections. With suitable representations for the Z_2 and Z_2' parities, the $SU(5)$ gauge symmetry is preserved on the O_B 3-brane, while it is broken down to the SM gauge symmetry on the O_B' 3-brane. To realize the gauge coupling relation in Eq. (2), we introduce the adjoint Higgs field in the **24**

representation on the O_B 3-brane. The gauge coupling relation in Eq. (2) can be generated via the suitable dimension-five operators after the adjoint Higgs field acquires the VEV [22–25]. We put the Higgs fields $\mathbf{5}_H = (T_u,\ H_u)$ and $\overline{\mathbf{5}}_{\overline{H}} = (T_d,\ H_d)$ in the bulk, and then T_u and T_d do not have zero modes due to the orbifold projections. In particular, the KK modes for T_u and T_d only have vector-like mass term via μ term. Thus, similar to the discussions in the above F-theory GUTs, we do not have the dimension-five proton decay problem. To forbid the dimension-six proton decay, we put the SM fermion superfields $\mathbf{10}_i$ and $\mathbf{10}'_i$ in the bulk with suitable Z_2 and Z'_2 parity assignments where $i = 1,\ 2,\ 3$. We obtain the SM fermions Q_i as zero modes from $\mathbf{10}_i$ while we obtain the SM fermions U_i^c and E_i^c as zero modes from $\mathbf{10}'_i$. Because the X and Y gauge bosons can not couple to both Q_i and U_i^c, we do not have the dimension-six proton decay problem via superheavy gauge boson exchanges.

In the orbifold $SO(10)$ models (for a concrete example, see Ref. [31]), the $SO(10)$ gauge symmetry is broken down to the Pati-Salam $SU(4)_C \times SU(2)_L \times SU(2)_R$ gauge symmetry via orbifold projections. With suitable representations for the Z_2 and Z'_2 parities, the $SO(10)$ gauge symmetry is preserved on the O_B 3-brane, while it is broken down to the Pati-Salam gauge symmetry on the O'_B 3-brane. To realize the gauge coupling relation in Eq. (2), we introduce the symmetric Higgs field in the $\mathbf{54}$ representation on the O_B 3-brane. The gauge coupling relation in Eq. (2) can be generated via the suitable dimension-five operators after the symmetric Higgs field acquires the VEV [22–25]. We put the Higgs field $\mathbf{10}_H = (T_u,\ H_u,\ T_d,\ H_d)$ in the bulk, and then T_u and T_d do not have zero modes due to orbifold projections. In particular, the KK modes for T_u and T_d only have vector-like mass term via μ term. Thus, similar to the discussions in the above F-theory GUTs and the orbifold $SU(5)$ models, we do not have the dimension-five proton decay problem. To forbid the dimension-six proton decay, we put the SM fermion superfields $\mathbf{16}_i$ and $\mathbf{16}'_i$ in the bulk with suitable Z_2 and Z'_2 parity assignments where $i = 1,\ 2,\ 3$. We obtain the left-handed SM fermions Q_i and L_i as zero modes from $\mathbf{16}_i$ while we obtain the right-handed SM fermions U_i^c, D_i^c, N_i^c and E_i^c as zero modes from $\mathbf{16}'_i$. Because the X and Y gauge bosons can not couple to both Q_i and U_i^c and the X' and Y' gauge bosons can not couple to both Q_i and D_i^c, we do not have the dimension-six proton decay problem via superheavy gauge boson exchanges.

Fourth, let us comment on the superheavy threshold corrections on the gauge coupling unification in our models. In the F-theory $SU(5)$ and $SO(10)$ models, we shall have the su-

perheavy threshold corrections from the Kaluza-Klein (KK) modes and heavy string modes. Because our unification scale is smaller than or equal to 10^{16} GeV, we do not have string theshfold corrections since the string scale is generic around 4×10^{17} GeV. Also, the KK modes can have masses around the GUT scale or higher, and then their effects on the gauge coupling unification can be negligible as well. Moreover, in the orbifold $SU(5)$ and $SO(10)$ models, we shall have the superheavy threshold corrections from the KK modes. Because the masses of the KK modes can not be larger than the GUT scale, we might have appreciable threshold corrections on the gauge coupling unification, which definitely deserves further detailed study. Thus, for simplicity, we assume that the KK mass scale is equal to the GUT scale in this paper.

V. CONCLUSION

Inspired by the string landscape and the unified gauge coupling relation in the F-theory GUTs and GUTs with suitable high-dimensional operators, we studied the canonical gauge coupling unification in the SM with high-scale supersymmetry breaking. In the SM with GUT-scale supersymmetry breaking, the gauge coupling unification can be achieved at about 5.3×10^{13} GeV, and the Higgs boson mass is predicted to range from 130 GeV to 147 GeV. In the SM with supersymmetry breaking scale from 10^4 GeV to 5.3×10^{13} GeV, gauge coupling unification can always be realized, and the corresponding GUT scale M_U is from 10^{16} GeV to 5.3×10^{13} GeV, respectively. Also, we obtained the Higgs boson mass from 114.4 GeV to 147 GeV. Moreover, the discrepancies among the SM gauge couplings at the GUT scale are less than about 4-6%. Furthermore, we presented the $SU(5)$ and $SO(10)$ models from the F-theory model building and orbifold constructions, and showed that there are no dimension-five and dimension-six proton decay problems even if $M_U \leq 5 \times 10^{15}$ GeV.

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